كلية الحاسبات والذكاء الإصطناعي

BS111

Probability and Statistics

Lecture 05

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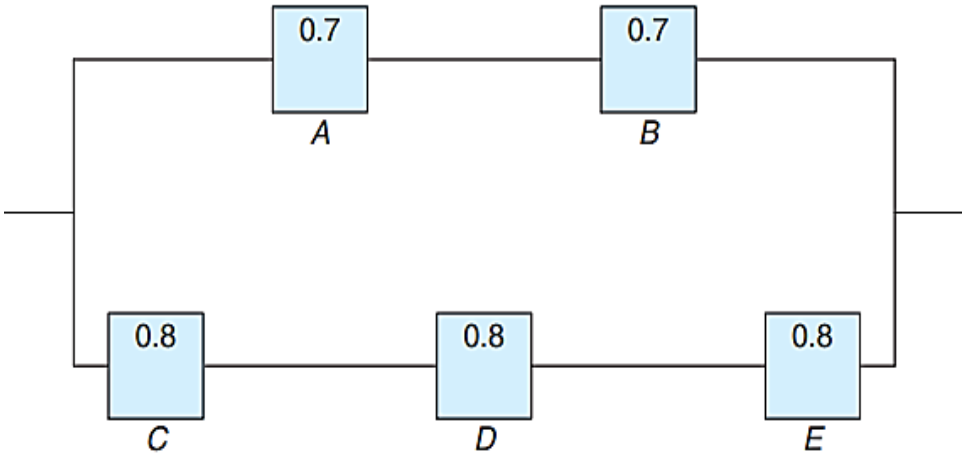
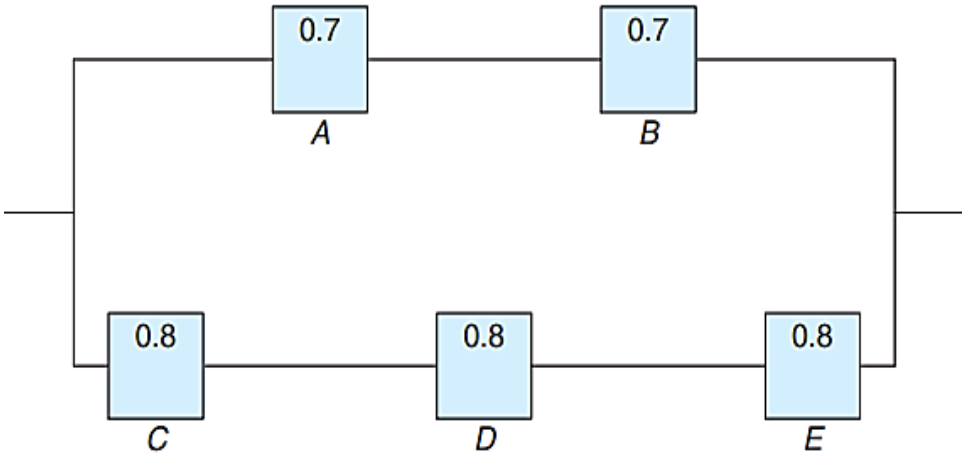
Quiz (1) – Q1

1. A circuit system is given in the following figure. Assume the components fail independently. What is the probability that the entire system works?



Quiz (1) – Q1

1. A circuit system is given in the following figure. Assume the components fail independently. What is the probability that the entire system works?



UP: (0.7) (0.7) = 0.49

Down: (0.8) (0.8) (0.8) = 0.512

$P(W) = 1 - [(0.51)\ (0.488)] = 0.75112$

Quiz (1) – Q2

2. If a multiple-choice test consists of 6 questions, each with 4 possible answers of which only 1 is correct. In how many ways can a student check off one answer to each question and get all the answers wrong?

Quiz (1) – Q2

2. If a multiple-choice test consists of 6 questions, each with 4 possible answers of which only 1 is correct. In how many ways can a student check off one answer to each question and get all the answers wrong?

Ways to get all the answers wrong $= (3)^6 = 729$

Quiz (1) – Q3

3. If A, and B are two independent events. And $P(A) = 0.2$, $P(A \cap B) = 0.06$, determine the following probabilities:

a) $P(A|B)$

b) $P(B|A)$

Quiz (1) – Q3

3. If A, and B are two independent events. And $P(A) = 0.2$, $P(A \cap B) = 0.06$, determine the following probabilities:

a) $P(A|B)$

$$P(A|B) = P(A) = 0.2$$

b) $P(B|A)$

$$P(B|A) = P(\mathrm{B}) = \frac{P(A \cap B)}{P(A)} = \frac{0.06}{0.2} = 0.3$$

Quiz (1) – Q4

4. In how many ways can 3 boys and 4 girls sit in a row?

Quiz (1) – Q4

4. In how many ways can 3 boys and 4 girls sit in a row?

$$\text{number of ways} = \frac{n!}{n_1!\, n_2!} = \frac{7!}{3!\, 4!} = 35$$

Quiz (1) – Q5

5. The likely location of a mobile device in the home is as follows:

Adult bedroom: 0.10, Child bedroom: 0.20,

Office: 0.40, Other rooms: 0.30

(a) What is the probability that a mobile device is in a bedroom?

(b) What is the probability that it is not in a bedroom?

Quiz (1) – Q5

5. The likely location of a mobile device in the home is as follows:

Adult bedroom: 0.10, Child bedroom: 0.20,

Office: 0.40, Other rooms: 0.30

(a) What is the probability that a mobile device is in a bedroom?

$= P(Adult\ bedroom \cup Child\ bedroom) = 0.10 + 0.20 = 0.30$

(b) What is the probability that it is not in a bedroom?

$= 1 - 0.30 = 0.70$

Quiz (1) – Q6

6. How many distinct permutations can be made from the letters of the word COMPUTER?

Quiz (1) – Q6

6. How many distinct permutations can be made from the letters of the word COMPUTER?

$n! = 8! = 40{,}320$

Quiz (1) – Q7

7. If a multiple-choice test consists of 6 questions, each with 4 possible answers of which only 1 is correct. In how many different ways can a student check off one answer to each question?

Quiz (1) – Q7

7. If a multiple-choice test consists of 6 questions, each with 4 possible answers of which only 1 is correct. In how many different ways can a student check off one answer to each question?

$= (4)^6 = 4{,}096$

Chapter 2: Random Variable

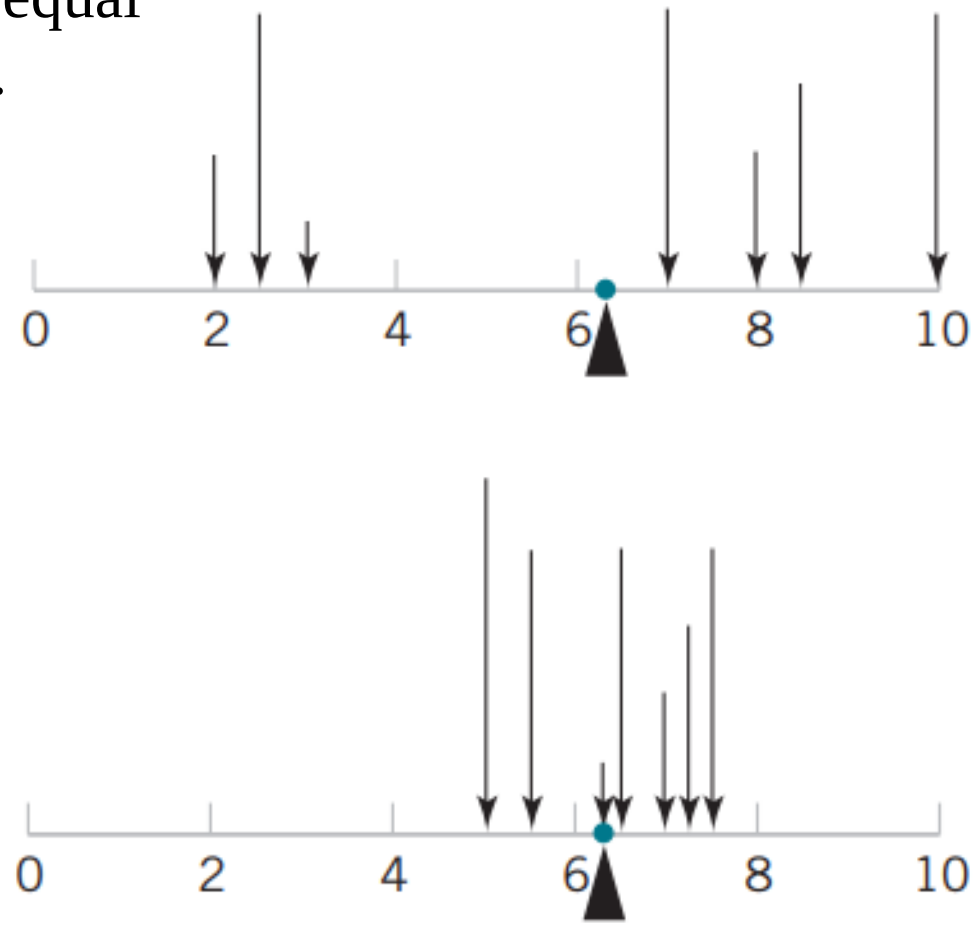
- Discrete Random Variables.
- Probability Mass Functions.
- Cumulative Distribution Functions.
- Discrete R.V. (Mean and Variance).
- Continuous Random Variables.
- Probability Density Functions.
- Continuous R.V. (Mean and Variance).
- Joint Probability Distributions.

Mean and Variance (1/15)

Two numbers are often used to summarize a probability distribution for a random variable *X*. The mean is a measure of the center or middle of the probability distribution, and the variance is a measure of the dispersion, or variability in the distribution.

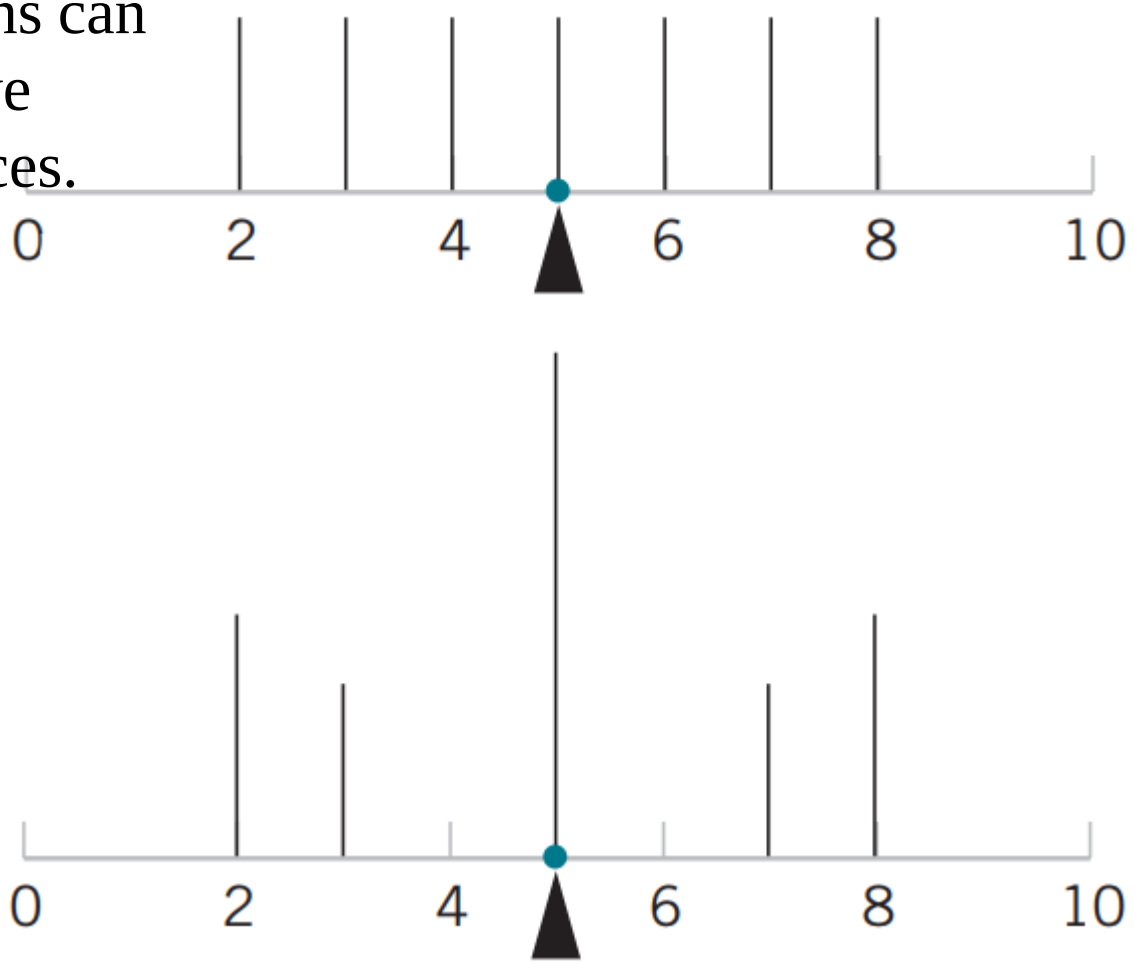
Mean and Variance (2/15)

Probability distributions with equal means but different variances.



Mean and Variance (3/15)

Two probability distributions can differ even though they have identical means and variances.



Mean and Variance (4/15)

Mean, Variance, and Standard deviation

The **mean** or **expected value** of the discrete random variable X, denoted as μ or $E(X)$, is

$$\mu = E(X) = \sum_x xf(x)$$

The **variance** of X, denoted as σ^2 or $V(X)$, is

$$\sigma^2 = V(X) = E(X-\mu)^2 = \sum_x (x-\mu)^2 f(x) = \sum_x x^2 f(x) - \mu^2$$

$$E(X^2) - \big(E(X)\big)^2$$

The **standard deviation** of X is $\sigma = \sqrt{\sigma^2}$.

Mean and Variance (5/15)

Example1

x	-2	-1	0	1	2
$f(x) = P(X = x)$	1/8	2/8	2/8	2/8	1/8

Find:

Determine the mean and variance of the random variable X

Mean and Variance (6/15)

Example1

x	-2	-1	0	1	2
$f(x) = P(X = x)$	1/8	2/8	2/8	2/8	1/8

Find:

Determine the mean and variance of the random variable X

Answer: (1/2)

$E(X) =$

$$\sum x_i P(x_i) = (-2)\left(\frac{1}{8}\right) + (-1)\left(\frac{2}{8}\right) + (0)\left(\frac{2}{8}\right) + (1)\left(\frac{2}{8}\right) + (2)\left(\frac{1}{8}\right)$$

$$= 0$$

Mean and Variance (6/15)

Example1

x	-2	-1	0	1	2
$f(x) = P(X = x)$	1/8	2/8	2/8	2/8	1/8

Answer: (2/2)

$$V(X) = E(X^2) - \big(E(X)\big)^2$$

$$E(X) = 0$$

$$E(X^2)$$

$$= \sum x_i^2 P(x_i) = (4)\left(\frac{1}{8}\right) + (1)\left(\frac{2}{8}\right) + (0)\left(\frac{2}{8}\right) + (1)\left(\frac{2}{8}\right) + (4)\left(\frac{1}{8}\right) = 1.5$$

$$V(X) = 1.5 - (0)^2 = 1.5, \qquad \text{Standard Deviation } (\sigma) = \sqrt{1.5}$$

Mean and Variance (7/15)

Example2:

A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.

Mean and Variance (8/15)

Example2 – Answer (1/9)

A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.

Let X represent the number of good components in the sample. **Then x can only take the numbers 0, 1, 2 and 3.**

Mean and Variance (8/15)

Example2 – Answer (2/9)

A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.

The probability distribution of X is

$$f(x) = \frac{\binom{4}{x}\binom{3}{3-x}}{\binom{7}{3}}, \qquad x = 0, 1, 2, 3.$$

Mean and Variance (8/15)

Example2 – Answer (3/9)

A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.

$$f(0) = P(X = 0) = \frac{\binom{4}{0}\binom{3}{3}}{\binom{7}{3}} = \frac{1}{35}$$

Mean and Variance (8/15)

Example2 – Answer (4/9)

A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.

$$f(1) = P(X = 1) = \frac{\binom{4}{1}\binom{3}{2}}{\binom{7}{3}} = \frac{12}{35}$$

Mean and Variance (8/15)

Example2 – Answer (5/9)

A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.

$$f(2) = P(X = 2) = \frac{\binom{4}{2}\binom{3}{1}}{\binom{7}{3}} = \frac{18}{35}$$

Mean and Variance (8/15)

Example2 – Answer (6/9)

A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.

$$f(3) = P(X = 3) = \frac{\binom{4}{3}\binom{3}{0}}{\binom{7}{3}} = \frac{4}{35}$$

Mean and Variance (8/15)

Example2 – Answer (7/9)

A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.

x	0	1	2	3
$f(x) = P(X = x)$	1/35	12/35	18/35	4/35

Mean and Variance (8/15)

Example2 – Answer (8/9)

Find the expected value of the number of good components in this sample.

x	0	1	2	3
$f(x) = P(X = x)$	1/35	12/35	18/35	4/35

$$E(X) = (0)\left(\frac{1}{35}\right) + (1)\left(\frac{12}{35}\right) + (2)\left(\frac{18}{35}\right) + (3)\left(\frac{4}{35}\right) = \frac{12}{7} = 1.7.$$

Mean and Variance (8/15)

Example2 – Answer (9/9)

$$E(X) = 1.7$$

Determine the variance of the random variable X

x	0	1	2	3
$f(x) = P(X = x)$	1/35	12/35	18/35	4/35

$$V(X) = E(X^2) - \big(E(X)\big)^2$$

$$E(X^2) = \sum \boldsymbol{x_i^2 P(x_i)} = \mathbf{0}\left(\frac{\mathbf{1}}{\mathbf{35}}\right) + (\mathbf{1})\left(\frac{\mathbf{12}}{\mathbf{35}}\right) + (\mathbf{4})\left(\frac{\mathbf{18}}{\mathbf{35}}\right) + (\mathbf{9})\left(\frac{\mathbf{4}}{\mathbf{35}}\right) = \frac{\mathbf{120}}{\mathbf{35}} = \mathbf{3.43}$$

$$V(X) = 3.43 - (1.7)^2 = 0.54, \qquad \text{Standard Deviation } (\sigma) = \sqrt{0.54} = 0.74$$

Mean and Variance (9/15)

For any constants a and b:

Mean

1. $E(a) = a\,, \qquad a \in \mathbb{R}$
2. $E(aX + b) = aE(X) + b\,, \qquad a, b \in \mathbb{R}$

Variance

1. $V(a) = 0\,, \; a \in \mathbb{R}$
2. $V(aX + b) = a^2V(X)\,, \qquad a, b \in \mathbb{R}$

Mean and Variance (10/15)

Example3:

A discrete random variable with $V(X) = 2.5$
Evaluate $V(2X + 1)$

Mean and Variance (11/15)

Example3 – Answer

A discrete random variable with $V(X) = 2.5$
Evaluate $V(2X + 1)$

$$V(aX + b) = a^2 V(X)\,, \qquad a, b \in \mathbb{R}$$

$$V(2X + 1) = 4V(X) = 4 \times 2.5 = 10$$

Mean and Variance (12/15)

Example4:

A discrete random variable with $E(X) = 2.5$
Evaluate $E(2X + 1)$

Mean and Variance (13/15)

Example4 – Answer

A discrete random variable with $E(X) = 2.5$
Evaluate $E(2X + 1)$

$$E(aX + b) = aE(X) + b\ , \qquad a, b \in \mathbb{R}$$

$$E(2X + 1) = 2E(X) + 1$$

$$E(2X + 1) = 2 \times 2.5 + 1 = 6$$

Mean and Variance (14/15)

Example5:

Let X is a random variable with mean 6 and variance 100. Consider another random variable Y such that $Y = 3X + 6$, evaluate the mean and variance of Y?

Mean and Variance (15/15)

Example5 – Answer

Let X is a random variable with mean 6 and variance 100. Consider another random variable Y such that $Y = 3X + 6$, evaluate the mean and variance of Y?

$$E(X) = 6 \quad , \quad V(X) = 100$$

$$E(Y) = E(3X + 6)$$

$$V(Y) = V(3X + 6)$$

Mean and Variance (15/15)

Example5 – Answer

Let X is a random variable with mean 6 and variance 100. Consider another random variable Y such that $Y = 3X + 6$, evaluate the mean and variance of Y?

$$E(X) = 6 \quad , \quad V(X) = 100$$

$$E(Y) = E(3X + 6) = 3E(X) + 6 = 3(6) + 6 = 24$$

$$V(Y) = V(3X + 6) = 9V(X) = 9(100) = 900$$

Continuous R. V. (1/3)

Continuous Random Variable:

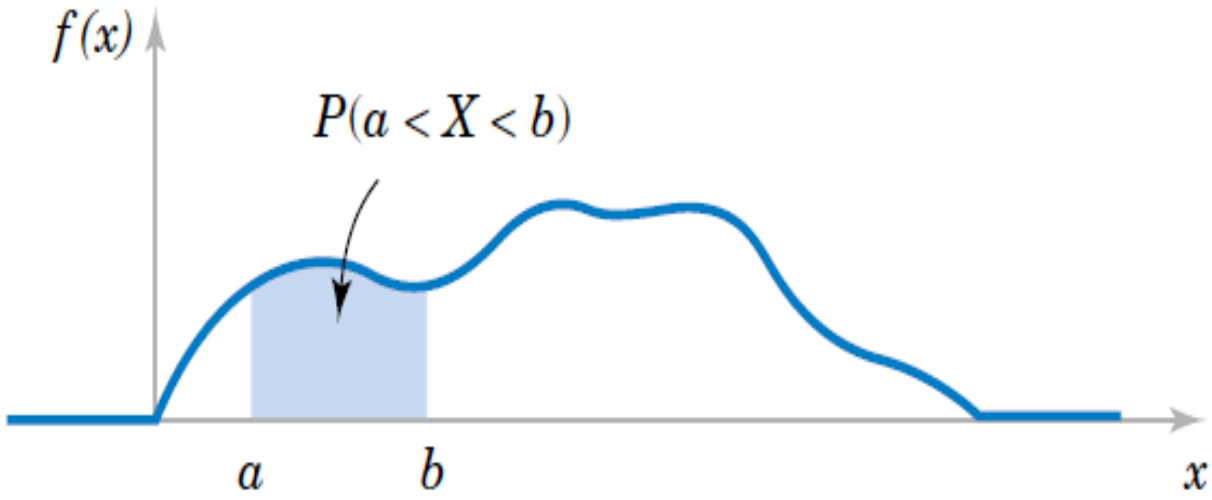
If the range space R_X of the random variable X is an interval or a collection of intervals, X is called a *continuous random variable*.

A continuous random variable has a probability of **0** of assuming *exactly* any of its values. Consequently, its probability distribution cannot be given in tabular form.

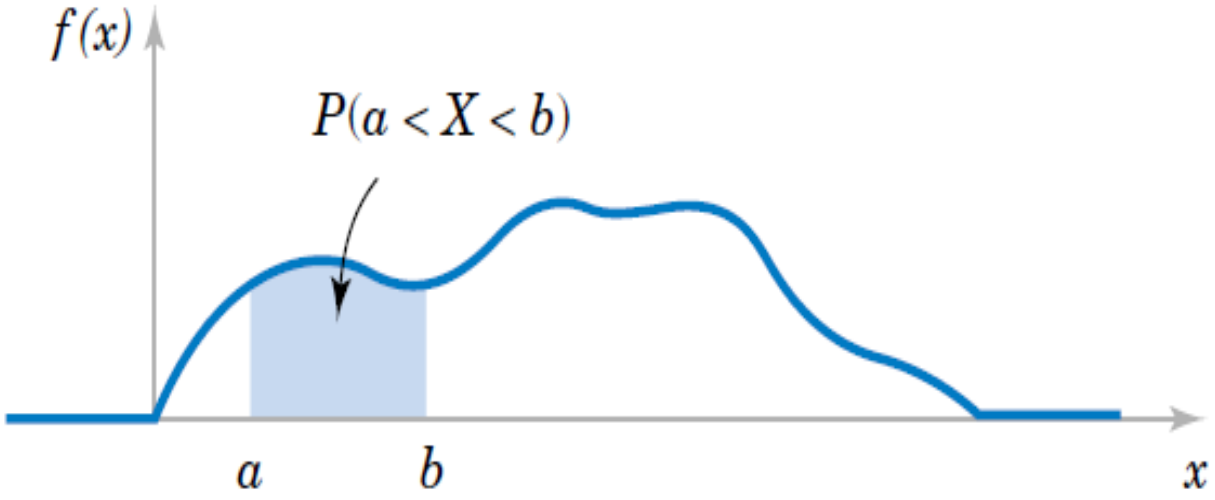
Continuous R. V. (2/3)

Example:

If we talk about the probability of selecting a person who is at least 163 centimeters but not more than 165 centimeters tall. Now we are dealing with an interval rather than a point value of our random variable.



Continuous R. V. (3/3)



If X is a **continuous random variable,** for any x_1 and x_2,

$$P(x_1 \le X \le x_2) = P(x_1 < X \le x_2) = P(x_1 \le X < x_2) = P(x_1 < X < x_2)$$

Prob. Density Functions (1/6)

Probability Density Function

For a continuous random variable X, a **probability density function** is a function such that

(1) $f(x) \geq 0$

(2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(3) $P(a \leq X \leq b) = \int_{a}^{b} f(x)\,dx =$ area under $f(x)$ from a to b

for any a and b

Prob. Density Functions (2/6)

Definite Integral:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\int_1^3 x^2\,dx$$

$$\int_1^3 x^2\,dx = \left[\frac{x^3}{3}\right]_1^3 = \left[\frac{3^3}{3}\right] - \left[\frac{1^3}{3}\right] = \left[9 - \left(\frac{1}{3}\right)\right] = \frac{26}{3}$$

Prob. Density Functions (3/6)

Example1:

Suppose that $f(x) = e^{-x}$ for $x > 0$
Check the probability density function, then determine the following probabilities:

1. $P(X < 1)$
2. $P(1 \leq X < 2.5)$
3. $P(X = 3)$
4. $P(X \geq 3)$

Prob. Density Functions (4/6)

Example1 – Answer (1/5)

Check the probability density function:

$$\int_0^{\infty} e^{-x}\, dx$$

$$\int_0^{\infty} e^{-x}\, dx = -e^{-x}\Big|_0^{\infty} = (-e^{-\infty}) - (-e^{0}) = 0 + 1 = 1$$

Prob. Density Functions (4/6)

Example1 – Answer (2/5)

1) $\boldsymbol{P(X < 1)}$

$$\boldsymbol{P(X<1) = \int_0^1 e^{-x}\, dx = -e^{-x}\Big|_0^1 = \left(-e^{-1}\right) - \left(-e^{0}\right)}$$

$$\boldsymbol{= -0.367879 + 1 = 0.632121}$$

Prob. Density Functions (4/6)

Example1 – Answer (3/5)

2) $\boldsymbol{P(1 \leq X < 2.5)}$

$$\boldsymbol{P(1 \leq X < 2.5) = \int_{1}^{2.5} e^{-x}\, dx = -e^{-x} \Big|_{1}^{2.5}}$$

$$\boldsymbol{= \left(-e^{-2.5}\right) - \left(-e^{-1}\right) = -0.082085 + 0.367879}$$

$$\boldsymbol{= 0.285794}$$

Prob. Density Functions (4/6)

Example1 – Answer (4/5)

3) $\boldsymbol{P(X = 3)}$

$$\boldsymbol{P(X = 3) = 0}$$

Prob. Density Functions (4/6)

Example1 – Answer (5/5)

4) $\boldsymbol{P(X \geq 3)}$

$$\boldsymbol{P(X \geq 3) = \int_{3}^{\infty} e^{-x}\, dx = -e^{-x}\Big|_{3}^{\infty} = (-e^{-\infty}) - (-e^{-3})}$$

$$\boldsymbol{= 0 + 0.049787 = 0.049787}$$

Prob. Density Functions (5/6)

Example2:

Suppose that the error in the reaction temperature, in °C (Celsius), for a controlled laboratory experiment is a continuous random variable X having the probability density function

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Verify that $f(x)$ is a density function.

(b) Find $P(0 < X \leq 1)$.

Prob. Density Functions (6/6)

Example2 – Answer (1/2)

Check the probability density function:

$$\int_{-1}^{2} \frac{x^2}{3}\, dx$$

$$\int_{-1}^{2} \frac{x^2}{3}\, dx = \frac{x^3}{9}\Big|_{-1}^{2} = \left(\frac{(2)^3}{9}\right) - \left(\frac{(-1)^3}{9}\right) = \frac{8}{9} + \frac{1}{9} = \mathbf{1}$$

Prob. Density Functions (6/6)

Example2 – Answer (2/2)

2) $\boldsymbol{P(0 < X \leq 1)}$

$$\boldsymbol{P(0 < X \leq 1)} = \int_0^1 \frac{x^2}{3}\, \boldsymbol{dx} = \frac{x^3}{9}\Big|_0^1$$

$$= \left(\frac{(1)^3}{9}\right) - \left(\frac{(0)^3}{9}\right) = \frac{1}{9} + \frac{0}{9} = \boldsymbol{\frac{1}{9}}$$

Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6gW9SgkmoxE5w9vQkID1_r-

Lecture #5: https://www.youtube.com/watch?v=8X8D20NdSK4&list=PLxIvc-MGOs6gW9SgkmoxE5w9vQkID1_r-&index=6

Thank You

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